DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3 **(4 marks)**

Solve $4e^{2x} = 81 - 5e^{2x}$ exactly for x.

Question 4 **(3 marks)**

Two independent samples of different sizes were taken from a population. The first sample had sample size n_1 and the second sample had sample size n_2. The sample proportions of males in the samples were the same. When 99% confidence intervals were calculated for each sample, it was found that the corresponding margin of error in the second sample was half that of the first sample.

What is the ratio of the two sample sizes, $\frac{n_2}{n_1}$?

Section Two: Calculator-assumed **65% (99 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 8 **(7 marks)**

Big Foods is a large supermarket company. The manager of Big Foods wants to estimate the proportion of households that do the majority of their grocery shopping in their stores.

A junior staff member at Big Foods conducted a survey of 250 randomly-selected households and found that 56 did the majority of their grocery shopping at a Big Foods store.

(a) (i) Calculate the sample proportion of households who did the majority of their grocery shopping at Big Foods. (1 mark)

(ii) Determine the 95% confidence interval for the proportion of households who do the majority of their grocery shopping at Big Foods. Give your answer to four decimal places. (3 marks)

(iii) What is the margin of error of the 95% confidence interval? Give your answer to four decimal places. (1 mark)

An independent research company conducted a large-scale survey of household supermarket preferences and estimated that the true proportion of households that conduct most of their grocery shopping at Big Foods was 0.17 (assume that this is indeed the true proportion).

(b) With reference to your answer to part (a)(ii), does this result suggest that the junior staff member at Big Foods made a mistake? (2 marks)

Question 10 **(12 marks)**

A survey in Western Australia was conducted on the popularity of a calculator known as Type A. Out of 1450 Year 12 students, the survey found that 986 students used the Type A calculator.

Determine the following.

(a) A 90% confidence interval, to three decimal places, for the proportion of Western Australian Year 12 students who use the Type A calculator. What assumption was made in calculating this interval? (3 marks)

(b) The margin of error in this confidence interval. (2 marks)

Another three surveys of Year 12 students were conducted on the use of Type A calculators across Australia.

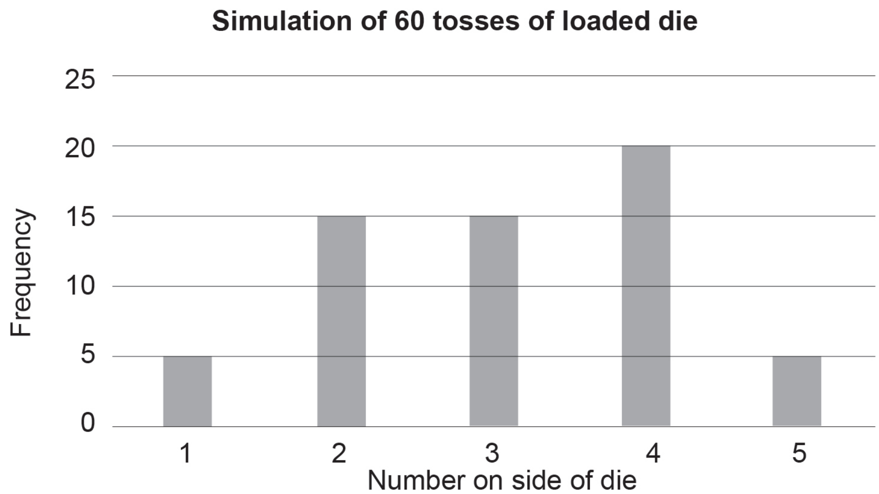
Survey 2	**Survey 3**	**Survey 4**
Type A usage 1772 out of 3221 Year 12 students	Type A usage 1021 out of 1566 Year 12 students	Type A usage 2203 out of 3221 Year 12 students

(c) Determine which of these surveys were more likely to have been taken outside of Western Australia. Justify your answer(s). (3 marks)

(d) Using the sample proportion of the survey at the start of the question, determine a sample size that will halve the margin of error for the proportion of Western Australian Year 12 students who use the Type A calculator, with a confidence of 90%. (4 marks)

Question 14 **(9 marks)**

The simulation of a loaded (unfair) five-sided die rolled 60 times is recorded with the following results.



(a) Calculate the proportion of prime numbers recorded in this simulation. (2 marks)

(b) Determine the mean and standard deviation for the sample proportion of prime numbers in 60 tosses, using the results above. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(c) It has been decided to create a confidence interval for the proportion of prime numbers using the simulation results on page 8. The level of confidence will be chosen from 90% or 95%. Explain which level of confidence will give the smallest margin of error. State this margin of error. (3 marks)

This simulation of 60 rolls of the die is performed another 200 times, with the proportion of prime numbers recorded each time and graphed.

(d) Comment briefly on the key features of this graph. (2 marks)

Question 18 **(11 marks)**

Alex is a beekeeper and has noticed that some of the bees are very sleepy. She takes a random sample of 320 bees and finds that 15 of them are indeed so-called *lullabees* that fall asleep easily.

(a) Calculate the sample proportion of lullabees. (1 mark)

(b) Determine a 90% confidence interval for the true proportion of lullabees, rounded to four decimal places. (3 marks)

(c) What is the margin of error in the above estimate? (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

It turns out that the true proportion of lullabees is 0.02.

(d) Now that Alex knows this, she decides to take a new sample.

(i) Suppose a new sample of 290 bees was taken. Given that the true proportion of lullabees is 0.02, what is the probability that the sample proportion in this new sample is at most 0.03? (3 marks)

(ii) If Alex takes a larger sample, will the above probability increase or decrease? Explain. (2 marks)

Question 20 **(14 marks)**

A chocolate factory produces chocolates of which 80% are pink. Each box of chocolates contains exactly 30 pieces.

(a) Identify the probability distribution of $X =$ the number of pink chocolates in a single box and also give the mean and standard deviation. (3 marks)

(b) Determine the probability, to three decimal places, that there are at least 27 pink chocolates in a randomly selected box. (3 marks)

Quality Control collects samples sizes of 20 boxes and counts the number of pink chocolates in total.

(c) Determine a 95% confidence interval for the proportion of pink chocolates in a sample of 20 boxes, using the assumption that 80% of chocolates in the sample are pink. (2 marks)

(d) Quality Control collects three samples and determines a 95% confidence interval each time. Determine the probability that only one of these intervals will **not** contain the true value 0.8 of the proportion of pink chocolates. (2 marks)

(e) Using your 95% confidence interval in part (c), determine the range in which the expected number of pink chocolates in a sample of 20 boxes would lie. (2 marks)

Quality Control counted the number of pink chocolates in five samples as shown below.

Sample	**1**	**2**	**3**	**4**	**5**
Number of pink chocolates	433	463	482	473	566

(f) Decide which samples lie outside the 95% confidence interval, if any. Justify. (2 marks)

Question 3 **(4 marks)**

Solve $4e^{2x} = 81 - 5e^{2x}$ exactly for x.

Solution
$4e^{2x} = 81 - 5e^{2x}$ $9e^{2x} = 81$ $e^{2x} = 9$ $\ln(e^{2x}) = \ln(9)$ $2x = \ln(9)$ $x = \dfrac{\ln(9)}{2}$
Specific behaviours
✓ collects exponential terms ✓ uses natural logs to simplify the equation ✓ uses log laws to simplify LHS of equation ✓ solves exactly for x

Question 4 **(3 marks)**

Two independent samples of different sizes were taken from a population. The first sample had sample size n_1 and the second sample had sample size n_2. The sample proportions of males in the samples were the same. When 99% confidence intervals were calculated for each sample, it was found that the corresponding margin of error in the second sample was half that of the first sample.

What is the ratio of the two sample sizes, $\dfrac{n_2}{n_1}$?

Solution
$z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n_2}} = \dfrac{1}{2} z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n_1}}$ $\dfrac{\hat{p}(1-\hat{p})}{n_2} = \dfrac{1}{4} \times \dfrac{\hat{p}(1-\hat{p})}{n_1}$ $\dfrac{n_2}{n_1} = 4$
Specific behaviours
✓ uses formula for margin of error to relate the two sample sizes ✓ simplifies equation by squaring both sides and cancelling ✓ rearranges to find ratio

Section Two: Calculator-assumed **65% (99 Marks)**

Question 8 **(7 marks)**

Big Foods is a large supermarket company. The manager of Big Foods wants to estimate the proportion of households that do the majority of their grocery shopping in their stores.

A junior staff member at Big Foods conducted a survey of 250 randomly-selected households and found that 56 did the majority of their grocery shopping at a Big Foods store.

(a) (i) Calculate the sample proportion of households who did the majority of their grocery shopping at Big Foods. (1 mark)

Solution
$\hat{p}=\dfrac{56}{250} = 0.224$
Specific behaviours
✓ calculates correct proportion

(ii) Determine the 95% confidence interval for the proportion of households who do the majority of their grocery shopping at Big Foods. Give your answer to four decimal places. (3 marks)

Solution
$\left(0.224-1.96\sqrt{\dfrac{0.224(1-0.224)}{250}},\ 0.224+1.96\sqrt{\dfrac{0.224(1-0.224)}{250}}\right)$ $(0.1723, 0.2757)$
Specific behaviours
✓ uses $Z = 1.96$ ✓ calculates confidence interval   ✓ rounds to four decimal places

(iii) What is the margin of error of the 95% confidence interval? Give your answer to four decimal places. (1 mark)

Solution
Either $E=1.96\sqrt{\dfrac{0.224(1-0.224)}{250}} = 0.0517$ or $E=\dfrac{0.2757-0.1723}{2} = 0.0517$
Specific behaviours
✓ calculates margin of error

An independent survey company conducted a large-scale survey of household supermarket preferences and estimated that the true proportion of households that conduct most of their grocery shopping at Big Foods was 0.17 (assume that this is indeed the true proportion).

(b) With reference to your answer to part (a)(ii), does this result suggest that the junior staff member at Big Foods made a mistake? (2 marks)

Solution
No. Only 95% of 95% confidence intervals are expected to contain the true proportion. It is possible that the survey and calculation by the junior staff member was performed appropriately, but happened to yield one of the 5% of confidence intervals that do not contain the true proportion.
Specific behaviours
✓ answers ‘No’ with a reference to part (a) ✓ justifies answer by saying that only 95% of intervals are expected to contain the true proportion

Question 10 **(12 marks)**

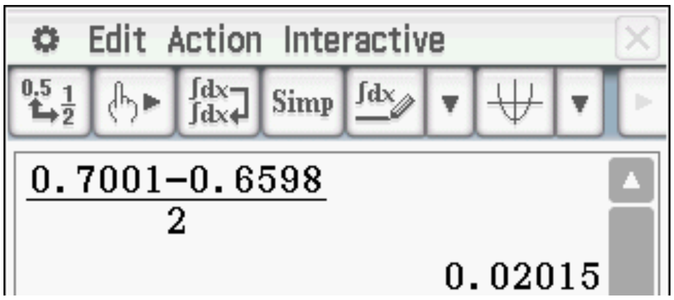
A survey in Western Australia was conducted on the popularity of a calculator known as Type A. Out of 1450 Year 12 students, the survey found that 986 students used the Type A calculator.

Determine the following.

(a) A 90% confidence interval, to three decimal places, for the proportion of Western Australian Year 12 students who use the Type A calculator. What assumption was made in calculating this interval? (3 marks)

Solution
$\hat{p} = \dfrac{986}{1450} = 0.68$ $s_p = \sqrt{\dfrac{0.68(1-0.68)}{1450}} = 0.01225$ $0.68 - 1.645(0.01225) \le p \le 0.68 + 1.645(0.01225)$ $0.6598 \le \hat{p} \le 0.7001$ $0.660 \le \hat{p} \le 0.700$ Assumes that sample proportions are a normal distribution.
Specific behaviours
✓ states that sample proportions form a normal distribution.   ✓ determines confidence interval   ✓ expresses interval rounded to three decimal places

(b) The margin of error in this confidence interval. (2 marks)

Solution

Specific behaviours
✓ uses confidence interval   ✓ determines margin of error

Another three surveys of Year 12 students were conducted on the use of Type A calculators across Australia.

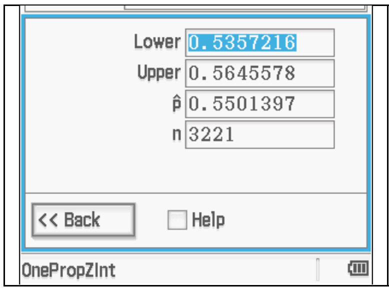
Survey 2	Survey 3	Survey 4
Type A usage 1772 out of 3221 Year 12 students	Type A usage 1021 out of 1566 Year 12 students	Type A usage 2203 out of 3221 Yr 12 students

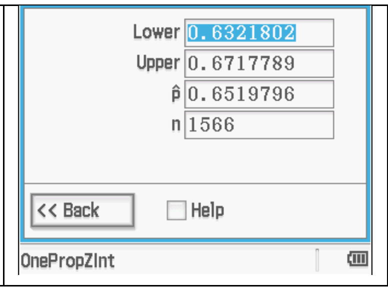
(c) Determine which of these surveys were more likely to have been taken outside of Western Australia. Justify your answer(s). (3 marks)

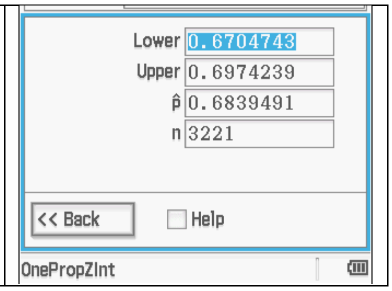
Solution

Survey 2 **Survey 3** **Survey 4**

The interval for Survey 2 does not overlap with interval in part (a),







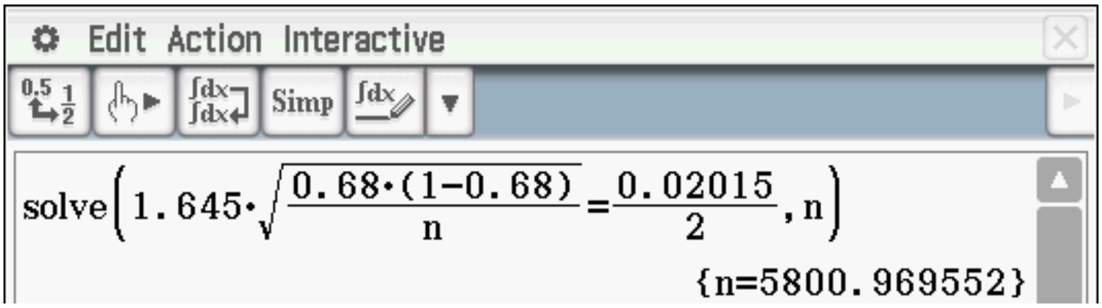
hence more likely to be taken outside of Perth.

Specific behaviours

✓ determines intervals for all three surveys
✓ compares to interval in part (a)
✓ uses an argument supported by confidence intervals

(d) Using the sample proportion of the survey at the start of the question, determine a sample size that will halve the margin of error for the proportion of Western Australian Year 12 students who use the Type A calculator, with a confidence of 90%. (4 marks)

Solution



Rounds up to $n = 5801$

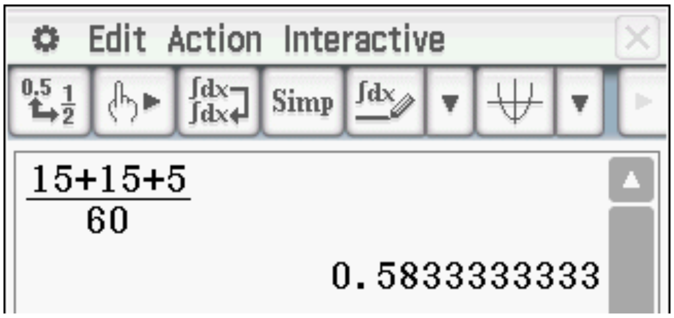
Specific behaviours

✓ uses the proportion from part (a)
✓ halves margin of error from part (b)
✓ sets up an equation to solve for sample size
✓ rounds sample size up to nearest integer

Question 14 **(9 marks)**

The simulation of a loaded (unfair) five-sided die rolled 60 times is recorded with the following results.

(a) Calculate the proportion of prime numbers recorded in this simulation. (2 marks)

Solution

Specific behaviours
✓ identifies the prime numbers ✓ determines proportion

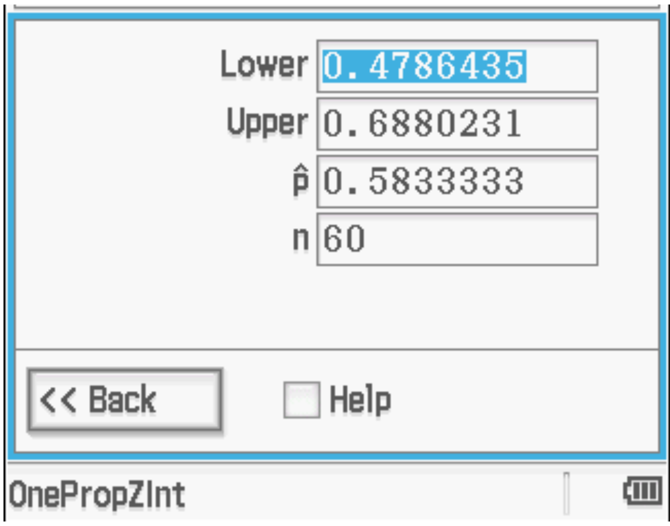
(b) Determine the mean and standard deviation for the sample proportion of prime numbers in 60 tosses, using the results above. (2 marks)

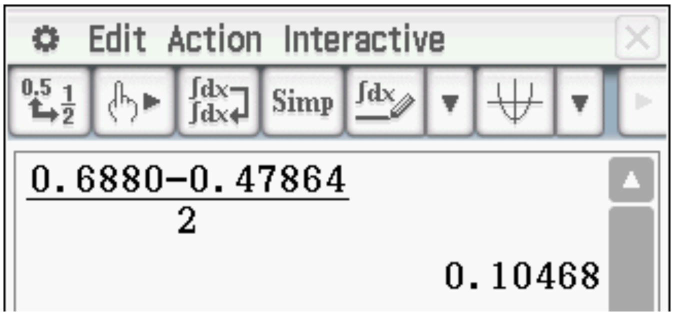
Solution
$\hat{p} = 0.58$ $s_x = \sqrt{\dfrac{0.58(1-0.58)}{60}} = 0.0637$
Specific behaviours
✓ determines the mean ✓ determines standard deviation

(c) It has been decided to create a confidence interval for the proportion of prime numbers using the simulation results on page 8. The level of confidence will be chosen from 90% or 95%. Explain which level of confidence will give the smallest margin of error. State this margin of error. (3 marks)

Solution

Smallest margin of error occurs for smallest confidence percentage 90%.
There is a trade-off between level of confidence and margin of error.





Specific behaviours

✓ uses 90% confidence
✓ states trade-off between confidence and margin of error
✓ determines margin of error

This simulation of 60 rolls of the die is performed another 200 times, with the proportion of prime numbers recorded each time and graphed.

(d) Comment briefly on the key features of this graph. (2 marks)

Solution

Graph takes the shape of a binomial distribution.
Approaches the shape of a normal distribution for large values of n.
Distribution is centred on 0.58.

Specific behaviours

✓ at least one of the descriptors above
✓ at least two descriptors above

Question 18 **(11 marks)**

Alex is a beekeeper and has noticed that some of the bees are very sleepy. She takes a random sample of 320 bees and finds that 15 of them are indeed so-called *lullabees* that fall asleep easily.

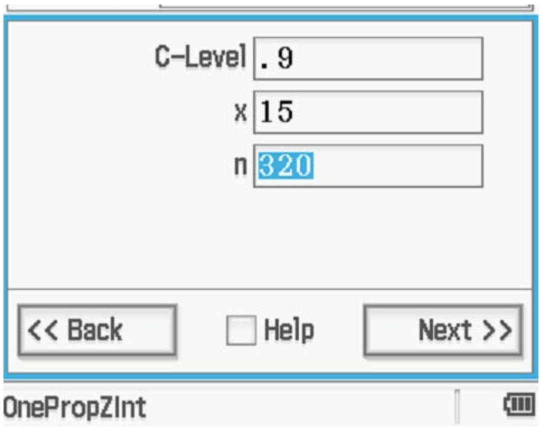
(a) Calculate the sample proportion of lullabees. (1 mark)

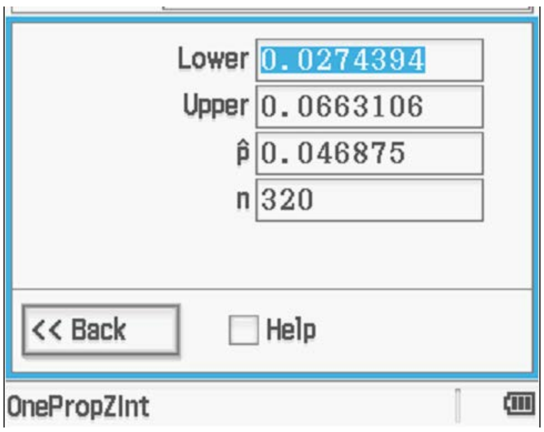
Solution
$\dfrac{15}{320} = 0.046875$
Specific behaviours
✓ calculates proportion

(b) Determine a 90% confidence interval for the true proportion of lullabees, rounded to four decimal places. (3 marks)

Solution

$$\left[\frac{15}{320} - 1.645\sqrt{\frac{\left(\frac{15}{320}\right)\left(1-\frac{15}{320}\right)}{320}}, \frac{15}{320} + 1.645\sqrt{\frac{\left(\frac{15}{320}\right)\left(1-\frac{15}{320}\right)}{320}}\right] \approx [0.0274, 0.0663]$$





Specific behaviours

✓ uses $Z \approx 1.645$,
✓ calculates confidence interval
✓ rounds to four decimal places

(c) What is the margin of error in the above estimate? (2 marks)

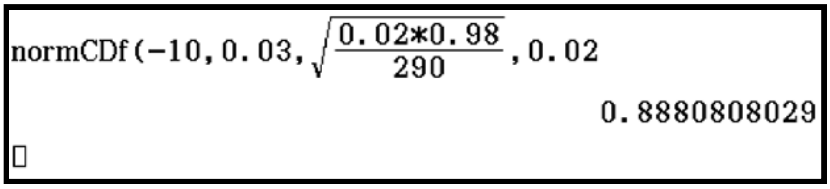
Solution
$1.645\sqrt{\dfrac{\left(\frac{15}{320}\right)\left(1-\frac{15}{320}\right)}{320}} = 0.0194$
Specific behaviours
✓ substitutes into formula ✓ calculates standard error

Question 18 (continued)

It turns out that the true proportion of lullabees is 0.02.

(d) Now that Alex knows this, she decides to take a new sample.

(i) Suppose a new sample of 290 bees was taken. Given that the true proportion of lullabees is 0.02, what is the probability that the sample proportion in this new sample is at most 0.03? (3 marks)

Solution
 i.e. a probability of approximately 0.89.
Specific behaviours
✓ CDF up to 0.03 ✓ determines standard deviation ✓ calculates probability

(ii) If Alex takes a larger sample, will the above probability increase or decrease? Explain. (2 marks)

Solution
Increase. The larger sample size will result in a smaller standard deviation. With a less dispersed distribution the required probability will increase.
Specific behaviours
✓ states increase and SD decreased ✓ states lower SD will give less dispersion and therefore higher probability

Question 20 **(14 marks)**

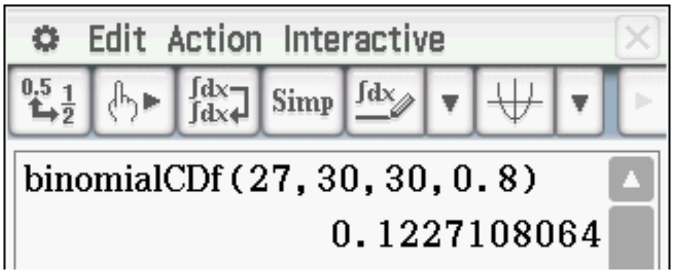
A chocolate factory produces chocolates of which 80% are pink. Each box of chocolates contains exactly 30 pieces.

(a) Identify the probability distribution of X = the number of pink chocolates in a single box and also give the mean and standard deviation. (3 marks)

Solution
$X \approx Bin\left(30, \frac{4}{5}\right)$ $u = 24$ $s = \sqrt{30\frac{4}{5}\left(1-\frac{4}{5}\right)} = 2.191$
Specific behaviours
✓ identifies binomial distribution   ✓ determines mean   ✓ determines standard deviation

(b) Determine the probability, to three decimal places, that there are at least 27 pink chocolates in a randomly selected box. (3 marks)

Solution



0.123 (rounded)

Specific behaviours
✓ uses binomial distribution with correct parameters   ✓ determines probability   ✓ rounds to three decimal places

Quality Control collects samples sizes of 20 boxes and counts the number of pink chocolates in total.

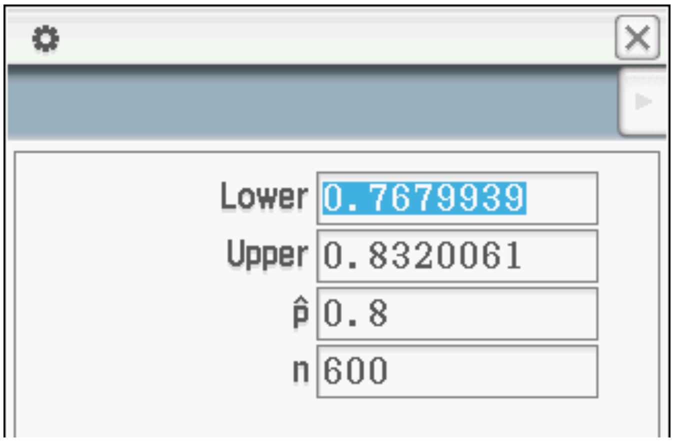
(c) Determine a 95% confidence interval for the proportion of pink chocolates in a sample of 20 boxes, using the assumption that 80% of chocolates in the sample are pink.
(2 marks)

Solution

$n = 20 \times 30 = 600$

$p = 0.8$

$x = 0.8 \times 600 = 480$



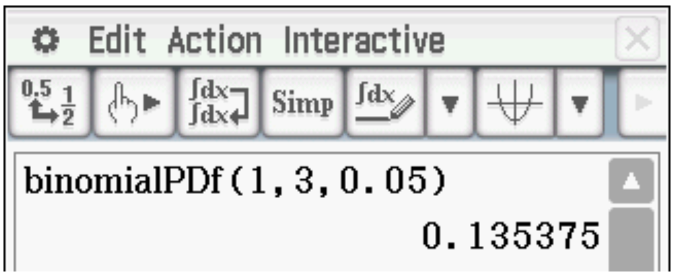
95% confidence interval $0.768 \le p \le 0.832$

Specific behaviours

✓ determines correct values for n, p and x
✓ determines confidence interval

(d) Quality Control collects three samples and determines a 95% confidence interval each time. Determine the probability that only one of these intervals will not contain the true value 0.8 of the proportion of pink chocolates (2 marks)

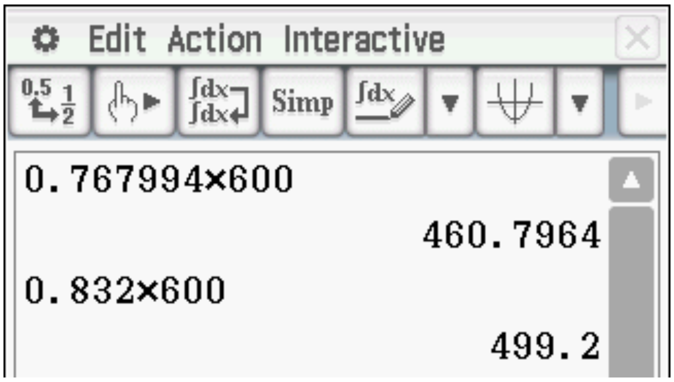
Solution



Specific behaviours

✓ uses a binomial distribution
✓ determines probability

(e) Using your 95% confidence interval in part (c), determine the range in which the expected number of pink chocolates in a sample of 20 boxes would lie. (2 marks)

Solution
   The range would lie between 460 and 499 pink chocolates.
Specific behaviours
✓ uses confidence interval ✓ multiplies proportion by total number of chocolates in sample

Quality Control counted the number of pink chocolates in five samples as shown below.

Sample	**1**	**2**	**3**	**4**	**5**
Number of pink chocolates	433	463	482	473	566

(f) Decide which samples lie outside the 95% confidence interval, if any. Justify. (2 marks)

Solution
Samples 1 and 5 lie outside the range in part (e), hence lie outside proportion interval.
Specific behaviours
✓ uses range of chocolates from part (e) ✓ presents an argument using confidence intervals